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ON THE STABILITY OF GYROSCOPIC STABILIZATION SYSTEMS

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We consider gyroscopic stabilization systems (GSS) on a fixed base with due regard to the fundamental properties of their elements [1] (elasticity, viscous friction in the material of the elastic bodies, transient responses in the electrical circuits of the system). We investigate the stability of the steady-state motion under parametric perturbations [2, 3].

1. Let $\varphi^1, \ldots, \varphi^r$ be the angles of natural rotation of the gyroscopes, let the gyromotors be asynchronous (induction) motors, and, in steady-state motions, let $\varphi^{\cdot i} = \omega^i =$ const $(i = 1, \ldots, r)$. The equations for GSS with synchronous gyromotors differ only by expressions for the moments around the rotors' axes [4, 5]. We denote by $q^1, \ldots, q^n$ the mechanical, and by $q^{n+1}, \ldots, q^{n+u}$ the electrical generalized coordinates, where $q^1, \ldots, q^l$ are the precession angles of the gyroscopes, $q^{l+1}, \ldots, q^m$ $(m = l + r)$ are the deviations of the φ^i from their values in steady-state motion, $q^{m+1}, \ldots, q^s$ $(s = m + l)$ are the angles of rotation of the rotors of the stabilizing motors, $q^{s+1}, \ldots, q^n$ are the deformations of the elastic elements, measured from their equilibrium position. In vector notation

$$\mathbf{q}_M = \left\|\begin{matrix} q^1 \\ \vdots \\ q^n \end{matrix}\right\|, \quad \mathbf{q}_1 = \left\|\begin{matrix} q^1 \\ \vdots \\ q^l \end{matrix}\right\|, \quad \mathbf{q}_2 = \left\|\begin{matrix} q^{l+1} \\ \vdots \\ q^m \end{matrix}\right\|, \quad \mathbf{q}_3 = \left\|\begin{matrix} q^{m+1} \\ \vdots \\ q^s \end{matrix}\right\|$$

$$\mathbf{q}_4 = \left\|\begin{matrix} q^{s+1} \\ \vdots \\ q^n \end{matrix}\right\|, \quad \mathbf{q}_E = \left\|\begin{matrix} q^{n+1} \\ \vdots \\ q^{n+u} \end{matrix}\right\|$$

The kinetic energy is $T = T_M + T_E$, where T_M is the kinetic energy of the mechanical part of the system with gyroscopes [6], T_E is the electromagnetic energy of the system [1] with the square matrices $a = \| a_{kj}(\mathbf{q}_M) \|$ and $L = \| L_{kj}(\beta_{ip}) \|$ of dimensions $n \times n$ and $u \times u$, respectively. We assume that the potential energy $\Pi(\mathbf{q}_4)$, of the elastic forces, the dissipative function $R_M(\mathbf{q}_M, \mathbf{q}_M^{\cdot})$ of the solid and viscous frictional forces, and the dissipative function $R_E(\mathbf{q}_E^{\cdot})$ of the currents, are holomorphic functions whose expansions start with terms of not less than second degree, with matrices $c = \| c_{kj} \|$, $b = \| b_{kj}(\mathbf{q}_M) \|$, $R = \| R_{kj} \|$, respectively.

We write the differential equations in A. V. Gaponov's form [1, 7]. Let in [1] the coefficients in the equations depend upon certain constant parameters $a^i = \alpha^i + \varepsilon^i$, the components of a χ-dimensional vector $\mathbf{a} = \boldsymbol{\alpha} + \boldsymbol{\varepsilon}$, where α^i is the rated value of the parameter, ε^i is its perturbation. Further, suppose that some perturbing forces of the form

$$\sum_{j=1}^{\varkappa} F_{kj}(\mathbf{q}_M, \mathbf{q}_M^{\cdot}, \mathbf{q}_E^{\cdot}, \mathbf{a})\gamma^j \qquad (k = 1, \ldots, n + u)$$

are acting on the system, where γ^j are perturbations of parameters, the components of a $\varkappa$-dimensional vector γ, whose rated values equal zero. We assume the holomorphicity of all the functions with respect to the collection of variables in a region A

$$\sum_{i=1}^{n}(q^i)^2 \leqslant A_1,\ \sum_{i=1}^{n}(q^{\cdot i})^2 \leqslant A_2,\quad \sum_{i=n+1}^{n+u}(q^{\cdot i})^2 \leqslant A_3$$

$$\sum_{i=1}^{\varkappa}(\varepsilon^i)^2 + \sum_{i=1}^{\varkappa}(\gamma^i)^2 \leqslant A_4;\quad A_i > 0$$

We take it that $F_{kj} = 0$ for $k = 1, \ldots, m,\ j = \nu + 1, \ldots, \varkappa,\ 0 \leqslant \nu < \varkappa$. since, from the point of view of influence on stability, we can distinguish two groups of parameters:

1) unessential parameters, small perturbations of which do not violate the system's stability (as shown below, there are all the a^i, $\gamma^{\nu+1}, \ldots, \gamma^{\varkappa}$ to which correspond, for example, the moments of inertia, the debalance of the platform, etc.);

2) essential parameters, small perturbations of which may lead to system instability (these are $\gamma^1, \ldots, \gamma^{\nu}$ to which correspond, for example, the debalance of the gyroscope's rotor, etc.). The perturbations of parameters of type 2 give rise to additional perturbing forces, at least, in the first m equations.

Suppose that the system of equations of a GSS (system (1) from [1] with the additions indicated) admits of a particular solution, defining the steady-state motion, which we take as the unperturbed one

$$\mathbf{q}_M^{\cdot} = 0,\quad \mathbf{q}_E^{\cdot} = \mathbf{q}_{E*}^{\cdot},\quad \mathbf{q}_E = \mathbf{q}_{E*}^{\cdot}t + \mathbf{q}_{E0},\quad \mathbf{q}_1 = \mathbf{q}_{1*},\quad \mathbf{q}_2 = \mathbf{q}_{20},\quad \mathbf{q}_3 = \mathbf{q}_{30},$$
$$\mathbf{q}_4 = 0,\quad \mathbf{a} = \boldsymbol{\alpha},\quad \boldsymbol{\gamma} = 0$$

Here $\mathbf{q}_{i_0}$ is an arbitrary real vector from A; the starred quantities satisfy the system

$$\sum_{ij=n+1}^{n+u} A_{ij}^{(k)} q^{\cdot i} q^{\cdot j} = 0\quad (k = m+1, \ldots, s),\qquad -\frac{\partial R_E}{\partial q^{\cdot k}} + E^k = 0\quad (k = n+1, \ldots, n+u)$$

Here, as in [1],

$$E^k = -\sum_{j=1}^{l} \omega_{kj} q^j + \ldots \quad (k = n+1, \ldots, n+l),$$

$$E^k = -\sum_{j=n+1}^{n+u} \Omega_{kj} q^{\cdot j} + \ldots \quad (k = n+l+1, \ldots, n+\mu),$$

$$E^k = E_*^{\,k}\ (k = n+\mu+1, \ldots, n+u),\quad \omega = \|\omega_{kj}\|,\quad \Omega = \|\Omega_{kj}\|,\quad l \leqslant \mu \leqslant u$$

where ω, Ω are constant matrices. We consider a GSS with commutator machines with fixed brushes ($\beta_{jk}^{\cdot} \equiv 0$).

2. Let us solve the problem of the stability of the unperturbed motion relative to $\mathbf{q}_M$, $\mathbf{q}_M^{\cdot}$, $\mathbf{q}_E^{\cdot}$ under parametric perturbations. The problem posed is equivalent to the problem of Liapunov stability with respect to $\mathbf{q}_M$, $\mathbf{q}_M^{\cdot}$, $\mathbf{q}_E^{\cdot}$, ε, $\boldsymbol{\gamma}$. The equations of unperturbed motion have the form

$$\frac{d}{dt} a\mathbf{q}_M^{\cdot} + (b^\circ + g^\circ)\,\mathbf{q}_M^{\cdot} = \mathbf{Q}_M' + \mathbf{Q}_M'' + F_M^\circ \boldsymbol{\gamma} \tag{2.1}$$

$$\frac{d}{dt} L\mathbf{q}_E^{\cdot} + B^\circ \mathbf{q}_3^{\cdot} = \mathbf{Q}_E' + \mathbf{Q}_E'' + F_E^\circ \boldsymbol{\gamma},\qquad \frac{d\mathbf{q}_M}{dt} = \mathbf{q}_M^{\cdot},\qquad \frac{d\varepsilon}{dt} = 0,\qquad \frac{d\boldsymbol{\gamma}}{dt} = 0$$

$$Q_M'(\mathbf{q}_E^{\cdot}, \mathbf{q}_M) = \left\| \begin{matrix} 0 \\ A^\circ \mathbf{q}_E^{\cdot} \\ -c^\circ \mathbf{q}_4 \end{matrix} \right\|, \qquad Q_E'(\mathbf{q}_E^{\cdot}, \mathbf{q}_M) = -\left\| \begin{matrix} \omega^\circ \mathbf{q}_1 + R_1 \mathbf{q}_E^{\cdot} \\ (R_2^\circ + \Omega^\circ)\mathbf{q}_E^{\cdot} \\ R_3^\circ \mathbf{q}_E^{\cdot} \end{matrix} \right\|$$

$$g = \| g_{kj} \|, \quad g_{kj} = \sum_{\nu=1}^{r} H^\nu \left(\frac{\partial a_k^{(\nu)}}{\partial q^j} - \frac{\partial a_j^{(\nu)}}{\partial q^k} \right), \qquad H^\nu = C_\nu \omega^\nu$$

$$B = \| B_{kj} \|, \quad B_{kj} = \sum_{i=n+1}^{n+u} B_{ki}^{(j)} q_*^{\cdot i}, \quad A = \| A_{kj} \|, \quad A_{kj} = \sum_{i=n+1}^{n+u} A_{ji}^{(k)} q_*^{\cdot i}$$

Here g, B, A are matrices of dimensions $n \times n$, $u \times l$, $l \times u$, respectively, F_M, F_E are submatrices of the ($(n + u) \times \varkappa$)-matrix $F = \| F_{kj} \|$, of dimensions $n \times \varkappa$, $u \times \varkappa$, respectively, R_1, R_2, R_3 are submatrices of matrix R, of dimensions $l \times u$, $(\mu - l) \times u$, $(u - \mu) \times u$, respectively, Q_M'' and Q_E'' are vector-valued functions, holomorphic in the collection of arguments, whose expansions start with terms of second degree (there may be first degree terms only in ε) and

$$F_M = \left\| \begin{matrix} F_1 \\ F_2 \end{matrix} \right\|, \quad Q_M'' = \left\| \begin{matrix} Q_1'' \\ Q_2'' \end{matrix} \right\|, \quad \gamma = \left\| \begin{matrix} \gamma_1 \\ \gamma_2 \end{matrix} \right\|$$

where F_1 is a matrix of dimension $m \times \varkappa$, Q_1'' and γ_1 are m-and ν-dimensional vectors, respectively. Here we have retained the previous notation for the perturbed variables; the values of the corresponding functions in unperturbed motion are denoted by an index zero.

In (2.1) we make a change of variables $\mathbf{q}_M = \mathbf{v}_M + \mathbf{p}_M$, $\mathbf{q}_E^{\cdot} = \mathbf{v}_E + \mathbf{p}_E$, and then $\mathbf{q}_i = \mathbf{v}_i + \mathbf{p}_i$ $(i = 1,2,3,4)$, respectively. Here $\mathbf{v}_M$ and $\mathbf{v}_E$ are vector-valued functions, holomorphic in ε, γ ($\mathbf{v}_2$ and $\mathbf{v}_3$ are null vectors), defined uniquely by the system

$$0 = \left\| \begin{matrix} A^\circ \mathbf{q}_E^{\cdot} \\ -c^\circ \mathbf{q}_4 \end{matrix} \right\| + Q_2''(0) + F_2^\circ \gamma, \qquad 0 = Q_E' + Q_E''(0) + F_E^\circ \gamma \tag{2.2}$$

if its determinant

$$\Delta = |\omega^\circ| \, |c^\circ| \left| \begin{matrix} A^\circ \\ R_2^\circ + \Omega^\circ \\ R_3^\circ \end{matrix} \right| \neq 0$$

In system (2.2), $Q_2''(0)$, $Q_E''(0)$ are the values of the corresponding functions when $\mathbf{q}_M^{\cdot} = \mathbf{q}_E^{\cdot} = \mathbf{q}_M = 0$. In the new variables Eqs. (2.1) are

$$\frac{d}{dt} a \mathbf{q}_M^{\cdot} - (b^\circ + g^\circ) \mathbf{q}_M^{\cdot} = \mathbf{P}_M' + \mathbf{P}_M'' + \left\| \begin{matrix} F_1^\circ \\ 0 \end{matrix} \right\| \gamma \tag{2.3}$$

$$\frac{d}{dt} L\mathbf{p}_E + B^\circ \mathbf{q}_3^{\cdot} = \mathbf{P}_E' + \mathbf{P}_E'', \qquad \frac{d\mathbf{p}_M}{dt} = \mathbf{q}_M^{\cdot}, \quad \frac{d\varepsilon}{dt} = 0, \qquad \frac{d\gamma}{dt} = 0$$

$$\mathbf{P}_M' = Q_M'(\mathbf{p}_M, \mathbf{p}_E), \qquad \mathbf{P}_E' = Q_E'(\mathbf{p}_M, \mathbf{p}_E)$$

where $\mathbf{P}_M''$ and $\mathbf{P}_E''$ are vector-valued functions, holomorphic in the collection of variables $\mathbf{p}_M$, $\mathbf{p}_E$, $\mathbf{q}_M^{\cdot}$, ε, γ, whose expansions do not contain terms of less than second degree. Let $\gamma_1 = 0$. Then, the following theorem is valid.

T h e o r e m . If except for m zero roots all the remaining roots of the characteristic equation of system (2.1) without parametric perturbations have negative real parts, then the unperturbed motion (the trivial solution of system (2.1)) is stable under parametric perturbations of type 1 relative to $\mathbf{q}_E^{\cdot}$, $\mathbf{q}_M^{\cdot}$, $\mathbf{q}_M$. As $t \to \infty$ every perturbed motion tends to one of the motions: $\mathbf{q}_M^{\cdot} = 0$, $\mathbf{q}_1 = \mathbf{U}_1 + \mathbf{v}_1$, $\mathbf{q}_2 = \mathbf{C}_2$, $\mathbf{q}_3 = \mathbf{C}_3$, $\mathbf{q}_4 = \mathbf{U}_4 + \mathbf{v}_4$, $\mathbf{q}_E^{\cdot} = \mathbf{U}_E + \mathbf{v}_E$, where $\mathbf{U}_1$, $\mathbf{U}_4$, $\mathbf{U}_E$ are holomorphic vector-valued functions defined by the system

$$0 = \left\| \begin{matrix} A^\circ \mathbf{p}_E \\ -c^\circ \mathbf{p}_4 \end{matrix} \right\| + \mathbf{P}_2''(\mathbf{q}_M^{\cdot} = 0), \qquad 0 = \mathbf{P}_E' + \mathbf{P}_E''(\mathbf{q}_M^{\cdot} = 0) \tag{2.4}$$

System (2.1) admits of a time-independent holomorphic Liapunov function

$$a_1 \mathbf{q}_M^{\cdot} + (b_1^\circ + g_1^\circ)\, \mathbf{q}_M + \varphi(\mathbf{q}_M^{\cdot},\ \mathbf{q}_E^{\cdot},\ \mathbf{q}_M, \varepsilon, \gamma) = \mathbf{N},$$

where a_1, b_1, g_1 are submatrices of the ($m \times n$)-matrices of the same designation ; φ, is an m-dimensional holomorphic function whose expansion starts with terms of not less than second order, vanishing at

$$\mathbf{q}_M^{\cdot} = 0,\ \ \mathbf{q}_E^{\cdot} = \mathbf{C}_E + \mathbf{v}_E,\ \mathbf{q}_1 = \mathbf{C}_1 + \mathbf{v}_1,\ \mathbf{q}_4 = \mathbf{C}_4 + \mathbf{v}_4$$

and at any $\mathbf{q}_2, \mathbf{q}_3, \varepsilon, \gamma$ from A; C_i, $\mathbf{N}$ are arbitrary constant vectors determined by the initial perturbations of the variables being considered.

Proof. We examine the stability of the trivial solution of (2.3) relative to all the variables $\mathbf{p}_M, \mathbf{p}_E, \mathbf{q}_M^{\cdot}, \varepsilon, \gamma$. We introduce the new variables

$$\mathbf{z} = a_1 \mathbf{q}_M^{\cdot} + (b_1^\circ + g_1^\circ)\, \mathbf{p}_M, \quad \mathbf{x}_1 = a \mathbf{q}_M^{\cdot}, \quad \mathbf{x}_2 = L \mathbf{p}_E, \quad \mathbf{x}_3 = \mathbf{p}_1, \quad \mathbf{x}_4 = \mathbf{p}_4$$

$$\mathbf{x} = \left\| \begin{matrix} \mathbf{x}_1 \\ \vdots \\ \mathbf{x}_4 \end{matrix} \right\|, \quad \mathbf{x}_1 = \left\| \begin{matrix} x^1 \\ \vdots \\ x^n \end{matrix} \right\|, \quad \mathbf{x}_2 = \left\| \begin{matrix} x^{n+1} \\ \vdots \\ x^{n+u} \end{matrix} \right\|, \quad \mathbf{x}_3 = \left\| \begin{matrix} x^{n+u+1} \\ \vdots \\ x^{n+u+l} \end{matrix} \right\|, \quad \mathbf{x}_4 = \left\| \begin{matrix} x^{n+u+l+1} \\ \vdots \\ x^{2n+u-m} \end{matrix} \right\| \tag{2.5}$$

Transformation (2.5) is nonsingular because its determinant Δ_1 is not zero. Indeed, to within sign

$$\Delta_1 = |a|\,|L|\,|\overset{\circ}{g}_{kj} + \overset{\circ}{b}_{kj}|_{k=1,\ldots,m}^{j=l+1,\ldots,s}$$

where by the theorem's hypothesis the characteristic equation $D(\lambda) = \lambda^m D_1(\lambda) = 0$ of the original system has only m zero roots, and to within sign

$$D_1(0) = |\omega^\circ|\,|c^\circ|\,|\overset{\circ}{g}_{kj} + \overset{\circ}{b}_{kj}|_{k=1,\ldots,m}^{j=l+1,\ldots,s} \left| \begin{matrix} A^\circ \\ R_2^\circ + \Omega^\circ \\ R_3^\circ \end{matrix} \right| \neq 0$$

In the new variable Eqs. (2.3) become

$$\frac{d\mathbf{z}}{dt} = \mathbf{Z}, \quad \frac{d\mathbf{x}_1}{dt} = -e^\circ \mathbf{x}_1 + \mathbf{X}_M' + \mathbf{X}_M'', \quad \frac{d\mathbf{x}_2}{dt} = -E^\circ \mathbf{x}_1 + \mathbf{X}_E' + \mathbf{X}_E''$$

$$\frac{d\mathbf{x}_3}{dt} = d_1 \mathbf{x}_1, \quad \frac{d\mathbf{x}_4}{dt} = d_4 \mathbf{x}_1, \quad \frac{d\varepsilon}{dt} = 0, \quad \frac{d\gamma_2}{dt} = 0 \tag{2.6}$$

$$\mathbf{X}_M' = \left\| \begin{matrix} 0 \\ A^\circ L^{-1} \mathbf{x}_2 \\ -c^\circ \mathbf{x}_4 \end{matrix} \right\|, \quad \mathbf{X}_E' = -\left\| \begin{matrix} \omega^\circ \mathbf{x}_3 + R_1^\circ L^{-1} \mathbf{x}_2 \\ (R_2^\circ + \Omega^\circ)\, L^{-1} \mathbf{x}_2 \\ R_3^\circ L^{-1} \mathbf{x}_2 \end{matrix} \right\|, \quad \mathbf{X}_M'' = \left\| \begin{matrix} \mathbf{X}_1'' \\ \mathbf{X}_2'' \end{matrix} \right\|$$

$$d = a^{-1}, \quad e = (b + g)\, d, \quad E = B d_3$$

Here d_1, d_2, d_3, d_4 are submatrices of d, of dimensions $l \times n$, $r \times n$, $(s-m) \times n$, $(n-s) \times n$, respectively ; $\mathbf{Z}$, $\mathbf{X}_M''$, $\mathbf{X}_E''$ are holomorphic vector-valued functions whose expansions start with terms of not less than second degree, which follows from the manner in which they were formed, and $\mathbf{Z}\,(\mathbf{x}_1 = 0) = 0$. Transformation is uniformly regular [8], i.e. the problem of stability relative to the old variables is equivalent to the problem of stability relative to the new variables $\mathbf{z}$, $\mathbf{x}$. By the theorem's hypotheses the characteristic equation of system (2.6) has, besides zero roots, roots only with negative real parts, since the first-approximation system for (2.6) can be obtained if in the first-approximation equations for (2.3) we replace the variables by means of a nonsingular linear transformation obtained from (2.5) by substituting a° for $a\,(\mathbf{v}_M + \mathbf{p}_M, \varepsilon)$. And for linear systems with constant coefficients the characteristic equation is invariant under non-singular linear transformations.

The algebraic system, obtained from (2.6) if we set $d\mathbf{x}_i/dt = 0$ $(i = 1, 2, 3, 4)$, determines, under the theorem's hypotheses, $\mathbf{x}_i = \mathbf{W}_i\,(\mathbf{z}, \varepsilon, \boldsymbol{\gamma}_2)$ as holomorphic functions of their own variables for sufficiently small values of their moduli, and $\mathbf{W}_1 = 0$, while the remaining satisfy the system

$$0 = \left\| \begin{matrix} A^\circ L^{-1}\mathbf{x}_2 \\ -c^\circ\mathbf{x}_4 \end{matrix} \right\| + \mathbf{X}_2''(\mathbf{x}_1 = 0), \qquad 0 = \mathbf{X}_E' + \mathbf{X}_E''(\mathbf{x}_1 = 0)$$

In the old variables system (2.4) corresponds to this system.

By the way in which it was formed we get that the vector-valued function $\mathbf{Z}\,(\mathbf{z}, \varepsilon, \boldsymbol{\gamma}_2, \mathbf{x} = \mathbf{W}) = 0$. Thus, by using the appropriate theorems in [9, 10], we can conclude that the trivial solution of (2.6) is stable relative to all the variables $\mathbf{z}, \mathbf{x}, \varepsilon, \boldsymbol{\gamma}_2$ and, as $t \to \infty$, any perturbed motion tends to one of the motions

$$\mathbf{z} = \mathbf{C}', \quad \mathbf{x}_1 = 0, \quad \mathbf{x}_i = \mathbf{W}_i\,(\mathbf{C}', \varepsilon, \boldsymbol{\gamma}_2) \quad (i = 2, 3, 4), \quad \varepsilon = \varepsilon_0, \quad \boldsymbol{\gamma}_2 = \boldsymbol{\gamma}_{20}$$

System (2.6) admits of a holomorphic integral

$$\mathbf{z} + \boldsymbol{\Phi}\,(\mathbf{z}, \mathbf{x} - \mathbf{W}, \varepsilon, \boldsymbol{\gamma}_2) = \mathbf{N}'$$

where $\boldsymbol{\Phi}$ is a holomorphic m-dimensional vector-valued function not containing terms lower than second degree and vanishing at $\mathbf{x} = \mathbf{W}$. Consequently, the trivial solution of (2.6) is stable relative to $\mathbf{z}, \mathbf{x}$ under parametric perturbations. We obtain the theorem's assertions by reverting to the original variables.

Note. We can show that the theorem is valid both for commutatorless machines as well as under the assumption that the gyromotors are alternators whose moments are [4, 5]

$$M^i = -c_i q^{l+i} - b_i \dot q^{l+i} + \xi^i(q^{l+i}, \dot q^{l+i}) \qquad (i = 1, \ldots, r' \leqslant r)$$

where ξ^i are holomorphic functions vanishing for $\dot q^{l+i} = 0$, $q^{l+i} = 0$.

Let us consider the influence of parametric perturbations of type 2 upon stability, i.e. let $\boldsymbol{\gamma}_1 = (\gamma^1 \ldots \gamma^\nu) \neq 0$. By transformation (2.5) we obtain a system in the new variables which differs from (2.6) only in the appearance of additional perturbing forces in the equations. Let $F_1^\circ \neq 0$, for example, $F_{11}^\circ \neq 0$. We set the last $\nu - 1$ components of $\boldsymbol{\gamma}_1$ equal to zero and we solve the problem of the stability of the trivial solution relative to $\mathbf{z}, \mathbf{x}, \varepsilon, \boldsymbol{\gamma}$. We take the scalar Liapunov function $V = (z^1 + \Phi^1)\gamma^1 F_{11}^\circ$, where $z^1 + \Phi^1 = N^1$ is an integral of system (2.6). The derivative $V^\bullet$ relative to the perturbed system

$$V^\bullet = (\gamma^1 F_{11}^\circ)^2 + (\gamma^1)^2\,\alpha\,(\mathbf{z}, \mathbf{x}, \varepsilon, \boldsymbol{\gamma})$$

$$\alpha\,(\mathbf{z}, \mathbf{x}, \varepsilon, \boldsymbol{\gamma}) = F_{11}^\circ \left[F_{11}' - F_{11}^\circ + \sum_{k=1}^{m} \frac{\partial \Phi^1}{\partial z^k} F_{k1}' + \sum_{i=1}^{n+u} \frac{\partial \Phi^1}{\partial x^i} F_{i1}' \right]$$

is a holomorphic function whose expansion starts with terms not less than first order. Thus $V^\bullet \geqslant 0$, and there exists a sufficiently small neighborhood wherein $V^\bullet = 0$ only for $\gamma^1 = 0$, i.e. for $V = 0$. Consequently, by Chetaev's theorem the trivial solution $\mathbf{z} = 0$, $\mathbf{x} = 0$, $\varepsilon = 0$, $\boldsymbol{\gamma} = 0$ is Liapunov unstable, i.e. the unperturbed motion is unstable under parametric perturbations of type 2.

Let us now consider γ^j $(j = 2, \ldots, \nu)$, i.e. perturbing forces of the form $F_{k2}\gamma^2 + \ldots + F_{k\nu}\gamma^\nu$, which we treat as constantly acting perturbations. The trivial solution is Liapunov unstable, therefore, by Gorshin's theorem [11], it is strongly unstable under constantly acting perturbations.

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